Close Wed: HW_5A, 5B, 5C (7.1,7.2,7.3)

See posting of several more integration by parts examples (with solutions).

Entry Task: Evaluate

$$\int_0^1 \sin^{-1}(x)dx = \int_0^1 \arcsin(x)dx$$

7.1 Integration by Parts (continued)

Summary: $\int u\,dv = uv - \int v\,du$

1. Pick u = ??. The rest is dv.
2. Differentiate to get du
   Integrate to get v.

Here are all the most common examples:

a) Products: xe^x, $x^2\cos(x)$, $x\sin(x)$
b) Logarithms: $\ln(x)$, $x^{10}\ln(x)$, …
c) Inv. Trig: $\sin^{-1}(x)$, $\tan^{-1}(x)$, …
d) Products: $e^x\sin(x)$, $e^x\cos(x)$

Example: (Never ending integration by parts and how to end it):

$$\int e^x \cos(x)\, dx$$

7.2 Trigonometric Integral Methods

Goal: To build up rules for integrate combinations of trig functions.

Basic motivating examples:
All of these could be done with substitution: What is *u*?

$$\int \sin^4(x)\cos(x)\,dx$$

$$\int \sin(x)\cos^3(x)\,dx$$

$$\int \tan^5(x)\sec^2(x)\,dx$$

$$\int \sec^6(x)\sec(x)\tan(x)\,dx$$

Idea: Use trig identities to turn a problem into a substitution problem like those above.

Tools

Essential Trig Identities:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)},$$
$$\sec(x) = \frac{1}{\cos(x)}, \csc(x) = \frac{1}{\sin(x)}.$$

$$\sin^2(x) + \cos^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$
$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$$

See my online posting (or Appendix D of your book) for a more general discussion of trig identities.

Add these to your table of integrals you know (an updated table is already on the website):

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x)\,dx$$
$$= \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$